INVESTIGATION OF EXTENDED BULB ANGLE SECTIONS UNDER COMPRESSION

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INVESTIGATION OF EXTENDED BULB ANGLE SECTIONS UNDER COMPRESSION

Part One: As Euler Columns

Part Two: As Stiffeners attached to sheet

Thesis by

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and

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Table of Contents

13679

Table of Contents (Cont'd)

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1. SUMMARY OF RESULTS

PART ONE

This investigation has established the following facts:

(a) Above an effective length/radius of gyration of about 78, extruded bulb angle sections follow the Euler curve.

(b) In the short column range the Straight Line Formula gives a more conservative value than the Johnson Parabolic Formula.

(c) From the experimental data the proportional limit for 24ST extruded sections was found to be 17000 lbs/in^2.

(d) When investigating extruded bulb angle sections used as columns for possible local plate failure, since the condition of support at the base of the angle is neither clamped nor simply supported, but an intermediate case, it is suggested that a value of K equal to 1.00 be used in the formula of Timoshenko for plate failure, when the ratio length/width is greater than four. Unfortunately, this suggestion is made on the basis of only two experimental plate failures.

II. STATEMENT OF PROBLEM

At the present time extrusions of various shapes are used in aeronautical construction, replacing sections made up of bent flat sheet. The properties and characteristics of the latter are known with sufficient accuracy; however, very little is known of the behavior of extrusions when acting alone as a column or as a stiffener attached to sheet.

It was considered advisable, before investigating the properties of sheet with attached extruded sections, to look into the behavior of the extruded sections when acting alone in compression.

When a column is acting alone under compression, failure may occur in any of the following ways:

(a) Failure as an Euler Column.

(b) Failure by local plate buckling.

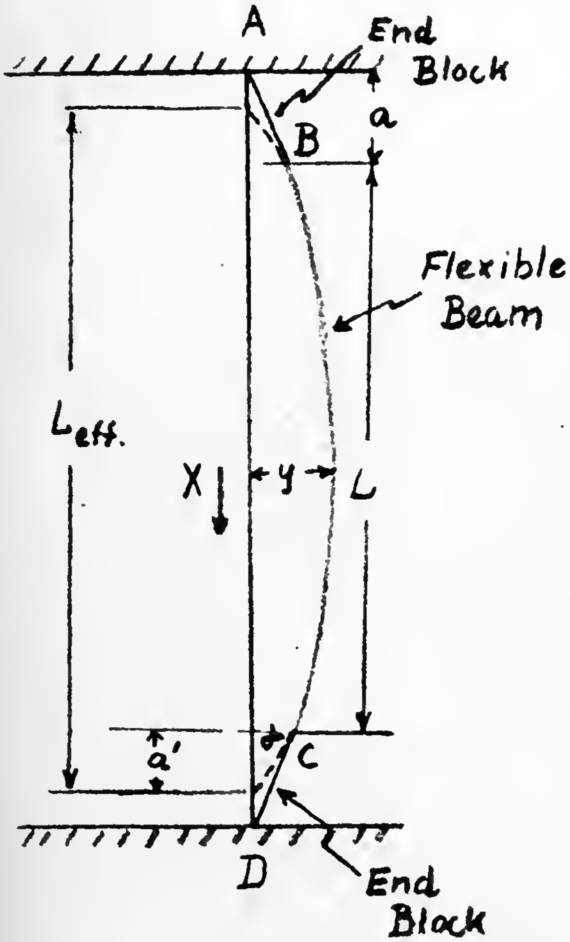
(c) Torsion failure.

Part One of this research is devoted to an investigation of the behavior of twelve representative extruded bulb angle sections when acting alone as columns.

Part Two of this research is devoted to an investigation of the behavior of extruded sections when acting as stiffeners riveted to sheet.

III. THEORETICAL CALCULATIONS

A development of the theory of effective lengths is reproduced from the Parr-Beakley thesis of 1935.(1) This method was originally deduced by Dr. von Kármán.



The equation for the bending of a beam is

$$EI \frac{d^2y}{dx^2} = -Py \qquad (1)$$

the solution of which is

$$y = A \cos\sqrt{\frac{P}{EI}}\, x \qquad (2)$$

Boundary Conditions:

AB and CD are supposed to be of infinite rigidity and hence are straight lines, therefore

$$\tan\alpha = -\left(\frac{dy}{dx}\right)_{x = \frac{L}{2}} = \alpha \qquad \text{(for small angles)}$$

$$Y_{x = \frac{L}{2}} = a\alpha = -a\left(\frac{dy}{dx}\right)_{x = \frac{L}{2}}$$

$$\text{or } A \cos\sqrt{\frac{P}{EI}}\,\frac{L}{2} = -a\left[-A\sqrt{\frac{P}{EI}}\ \sin\sqrt{\frac{P}{EI}}\ \frac{L}{2}\right]$$

$$\text{or } \operatorname{Cot}\sqrt{\frac{P}{EI}}\,\frac{L}{2} = a\sqrt{\frac{P}{EI}} \qquad (3)$$

and equation (3) gives the exact value of the P_{Euler} for the case considered.

For a = o we obtain $\cot\sqrt{\frac{P}{EI}}\ \frac{L}{2} = 0$

$$\text{i.e} \sqrt{\frac{P}{EI}}\ \frac{L}{2} = \frac{\pi}{2} \quad \text{or } P = \frac{\pi^2 EI}{L^2} \tag{4}$$

Putting $\sqrt{\frac{P}{EI}}\ \frac{L}{2} = Z$ equation (3) may be rewritten

$$Z \tan Z = \frac{L}{2a} \tag{5}$$

$$\text{and} \sqrt{\frac{P}{EI}} = \frac{2Z}{L}$$

$$\text{or} \quad P = \frac{EI \cdot 4Z^2}{L^2} \tag{6}$$

and comparing equation (6) with equation (4) it is seen that the effective length

$$L_{eff} = \frac{L\pi}{2Z} \tag{7}$$

therefore:

$$a' = \frac{1}{2}(L_{eff} - L) = \frac{L}{2}\left(\frac{\pi}{2Z} - 1\right)$$

or substituting from equation (5):

$$a' = a\left(\frac{\pi}{2} - Z\right)\tan Z \tag{I}$$

and from (5)

$$\frac{a}{L} = \frac{1}{2\ Z \tan Z} \tag{II}$$

and so, if we take different values of Z we can calculate corresponding values of a' and $\frac{a}{L}$.

Z	tan Z	a/L	a'/a
$\frac{\pi}{2}$	∞	0	1.0
$\frac{17\pi}{36}$	11.43	0.029	0.999
$\frac{5\pi}{12}$	3.732	0.102	0.982
$\frac{\pi}{3}$	1.732	0.275	0.907

It may be seen from the table that even where the rigid block equals 27.5% of the length of the beam proper, 90.7% of the length of the rigid portion is to be added to the length of the beam to give the effective length.

We will now apply this theory to the research problem at hand. From the data above, plot a curve of a/L vs a'/a (Fig.1). We obtain:

a	L	a/L	a'/a (from curve)
1.125	22	0.0512	0.996
1.125	16.5	0.0682	0.992
1.125	11	0.1023	0.982
1.125	5.5	0.2045	0.9417

To find $L_{effective}$:

$$L_{eff} = L + 2a \cdot a'/a$$
$$= 22 + 2.25 \times .996 = 24.24''$$
$$= 16.5 + 2.25 \times .992 = 18.73''$$
$$= 11 + 2.25 \times .982 = 13.21''$$
$$= 5.5 + 2.25 \times .9417 = 7.62''$$

Knowing the column effective length it is now possible to proceed with a study of the theoretical curves of failure.

In the long column range we shall use the Euler formula:

$$\sigma_{cr} = \frac{C \pi^2 E}{\left(\frac{L_{eff}}{\rho}\right)^2} \qquad (1)$$

where $C = 1.0$

In the short column range both the Johnson Parabolic formula and the "Straight-Line" formula were plotted, (Fig.2)

$$\sigma_{cr} = \sigma_y - \frac{\sigma_y{}^2 \left(\frac{L_{eff}}{\rho}\right)^2}{4 \pi^2 E} \qquad (2)$$

$$\sigma_{cr} = 48000 - 400 \left(\frac{L_{eff}}{\rho}\right) \qquad (3)$$

Investigations by W.L. Howland at California Institute of Technology have placed the proportional limit of aluminum alloys at 19000 lbs/in^2 (24ST). Our investigations offer

an opportunity to check this value, since at this stress the experimental points should separate from the Euler curve.

This has been carried out approximately as follows: Knowing the value of L_{eff}/ρ at which the experimental points leave the Euler curve we may solve for

$$\sigma_{p.1} = \frac{\pi^2 E}{\left(\frac{L_{eff}}{\rho}\right)^2}$$

$$\sigma_{p.1} = \frac{\pi^2 \times 10{,}500{,}000}{(78)^2}$$

$$\sigma_{p.1} = \underline{17050} \text{ lbs/in}^2$$

<u>Investigation into plate failure of the plain angle leg.</u>

<u>From Timoshenko:</u>(2) <u>Specimen 8477</u>

$$\sigma_{cr} = k\,\sigma_e$$

$$\sigma_e = \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2$$

$$\frac{a}{b} = \frac{7.62}{1.25} = 6.1$$

$$t = \frac{1}{2}\left[\frac{3}{32} + \frac{1}{16}\right] = \underline{0.078} \qquad \frac{t}{b} = 0.062$$

$$\sigma_e = \frac{\pi^2 \times 10{,}500{,}000}{12 \times .93} (.062)^2 = \underline{35800}$$

If we assume three sides simply supported and the fourth free, we get

$$k \doteq 0.5$$

$$\sigma_{cr} = 0.5 \times 35800 = \underline{17900} \text{ lbs/in}^2$$

If we assume two opposite sides simply supported, the third built in, and the fourth free, we get,

$$k = 1.33$$

$$\sigma_{cr} = 1.33 \times 35800 = 47600 \text{ lbs/in}^2$$

Now experimentally we find a σ_{cr} of 37400 lbs/in^2 and therefore we see that we have neither simple support, nor rigid clamping at the side which is supported by the other leg, but as we should expect, something between the two. If we use this σ_{cr} we may find the experimental value of K for this case:

$$K = \frac{37400}{35800} = \underline{1.044}$$

Investigation of Specimen 8478 for plate failure.

$$t = .062$$

$$t/b = \frac{.062}{1.0} = \underline{0.062} \qquad (t/b)^2 = \underline{0.00384}$$

$$a/b = \frac{7.62}{1.0} = 7.62$$

$$\sigma_{cr} = k\sigma_e$$

$$\sigma_e = \frac{\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 = \underline{35700}$$

As in the preceding case we may solve for the experimental value of K. In this case $\sigma_{exp.} = \underline{34400}$.

$$k = \frac{34400}{35700} = \underline{0.965}$$

Unfortunately we experienced only two plate failures. However, if we use the mean value of K determined above, we can check other specimens (which failed as columns) to see if the plate failure stress is higher, thus checking the possibility of that stress being critical.

If we check specimen 10282, which failed as a column at a stress of 15780 lb/in^2, for plate failure with our experimental value of K equal to 1.0, we find:

$t = .04$

$b = .50$

$K = 1$

$$\sigma_{cr} = \frac{1 \times \pi^2 \times E}{12(1-\mu^2)}\left[\frac{.04}{.5}\right]^2$$

$$\sigma_{cr} = 60700 \text{ lbs/in}^2$$

That the stress for plate failure is nearly four times the stress for column failure.

Timoshenko[2] gives values of K for a condition when three sides are simply supported and the fourth free, and for a condition when two opposite sides are simply supported, the third side built-in, and the fourth free. For a bulb angle section neither condition describes the actual condition of the side at the base of the angle because, since it is attached to the other leg, it cannot be considered hinged, nor can it be considered fixed, since the latter would imply complete rigidity which is not the case. Apparently then the condition that describes the support of the side at the base of the angle lies somewhere between these two conditions. Our average experimental value of K confirms this assumption.

It is suggested that when bulb angle sections are used alone as columns under compression and pending further investigation, the value of K (as used by Timoshenko in Strength of Materials, Vol. II) be taken as 1.00.

IV. EXPERIMENTAL INVESTIGATION

(a) Description of specimens:

The various bulb angle sections used in aircraft construction by several of the major aircraft manufacturers, were examined and from the lot a representative group of twelve was chosen for this investigation. All of the specimens chosen were 24ST alloy. After giving consideration to current bulkhead spacings between which extruded sections are used as stiffeners and at the same time taking into account the extension into the Euler long and short column ranges, column lengths of 22, $16\frac{1}{2}$, 11, and $5\frac{1}{2}$ inches were chosen for test purposes. In preparing the specimens it was essential that the ends be milled square. It should be noted that, since bulb angle sections are extrusions, their dimensions may vary considerable from those given in the specifications. It was found necessary to check all dimensions and to recompute all the section properties. These properties may vary as much as ten percent from those given in the manufacturer's specifications, (Table 1).

(b) Description of Apparatus:

In order to obtain a true hinged end condition of the columns under test, an end fitting was constructed. A half inch ball-bearing was sunk into the base plate of this fitting, and bears on a circular hardened plate which in turn rests upon the base plates of the compression cage. The ends of the bulb angle are clamped into the fittings where adjustments in

two directions are provided, enabling the centroid of the bulb angle to be located directly over the point of tangency of the ball-bearing.

The adjustments in the fittings enable any eccentricity present in the set-up to be removed. Two dial gauges are used to determine if any eccentricity is present. One gauge is mounted on a bracket attached to the cage, the other is held by a rigid bar mounted on flexible tabs which in turn are fixed between the circular end plates and the ends of the compression cage. The dial gauge plungers bear on the sides of the bulb angle at the midpoint of the column. Since any restraint of the column is most undesirable it was necessary to remove the plunger main springs of the dial gauges, having only the hair spring acting.

The detail photographs included in the appendix show clearly the construction of the end fittings and the method of setting up the specimens.

(c) Testing Procedure:

The bulb angle is mounted in the end fittings and placed in the compression cage. The circular bearing plates are then inserted between the balls and the base plates of the compression cage. A slight load is applied to the column in order to hold it in the machine and then it is placed approximately vertical by means of a level. The dial gauges are then attached and the load increased, a change in the readings of the gauges

denotes the presence of initial eccentricity which is removed by the adjustments in the end fittings. When the load can be increased to about a third of the anticipated final load without a change in the dial gauge readings it is assumed that all initial eccentricity has been removed. The dial gauges are now removed and the load increased steadily until failure. The tests were made on two Riehle Brothers testing machines, the longer columns in the 3000 pound capacity unit and the shorter columns in the 30,000 pound capacity unit. The type of failure, load, and description of the specimen were recorded.

V. DISCUSSION OF EXPERIMENTAL RESULTS.

In any open section tested as a column under compression three types of failures may be encountered, i.e. column, plate and torsion. Only two of these three possible types of failure occurred in this investigation, column and plate. Of the forty-eight specimens tested, forty-five failed as columns, two ($5\frac{1}{2}$ inch length) as plate failures, and one ($5\frac{1}{2}$ inch length) as a column failure of the bulb alone.

The plain legs of the two specimens which failed from plate failure show a relatively thin section. The theoretical calculations of section III show this type of failure to be critical for these specimens. It is interesting to note that the one specimen of the series whose bulb was on the reverse side, that is, outside of the opening between legs, failed as a column failure of the bulb alone.

During the tests great care was exercised in order to obtain, as nearly as possible, a fixity of unity, and to remove, as far as possible, any initial eccentricities. It appears from the attached curves that we were successful within the limits of experimental error, (Fig. 2).

Unfortunately the opportunities were very limited as regards the investigation of plate failure. However, the two specimens which so failed have been analyzed under the section devoted to theoretical calculations.

No torsion failures were encountered in the specimens tested.

VI. CONCLUSIONS.

The material with which this part of the research problem was performed was not entirely satisfactory, due to the fact that the investigators had no control over the parameters involved. The bulb angle section, being an extrusion, had to be taken as it could be obtained from the Industry. While probably not warranted, it would be desirable from a research point of view, to have a special series of dies, thus permitting a series of specimens in which one dimension could be varied holding the others constant. This would permit a more systematic study of the effect of changes in the parameters and should enable a prediction of an optimum cross-section.

From the investigation as carried out the following conclusions may be listed:

(1) Above a value of $L_{effective}/\rho = 78$ the Euler curve is followed closely, (Fig.2).

(2) Below this value of L_{eff}/ρ the straight line formula ($\sigma = 48000 - 400 \quad L_{eff}/\rho$) appears to approximate the experimental points more closely than the Johnson parabolic formula.

(3) The proportional limit for 24ST extrusions is approximately 17000 lbs/sq.in.

(4) When investigating the plain leg of the bulb angle section for local plate failure the following experimental

value of K for use in the formulae of Timoshenko more nearly describes the condition of support at the base of the angle.

K (Timoshenko) = 1.00

PART ONE, TABLE I

Comparison of dimensions and areas of extruded bulb angle sections as actually measured and computed, with those taken from the blue prints of the manufacturer.

As measured and computed

Section (Alcoa No.)	A	B	R	T_1	T_2	Area
8477	1.125	1.250	.125	.138	.087	.2774
8476	1.500	.687	.113	.051	.055	.1501
8478	1.125	1.000	.109	.073	.061	.1679
K-10266	1.000	.688	.095	.059	.065	.1220
10265	.875	.500	.094	.052	.056	.0931
10282	.750	.500	.067	.047	.042	.0647
3046	1.500	.568	.082	.052	.058	.1252
5436	1.500	.996	.156	.104	.079	.2470
12224	1.094	.625	.125	.105	.108	.1709
K-4200	1.094	.624	.118	.078	.084	.1334
K-766	.879	.499	.095	.062	.067	.0856
12678	.507	.445	.065	.044	.044	.0402

As taken from the blueprints of the manufacturer

Section (Alcoa No.)	A	B	R	T_1	T_2	Area
8477	1.125	1.250	.125	.125	.0625	.256
8476	1.500	.687	.109	.051	.051	.144
8478	1.125	1.000	.109	.072	.062	.168
K-10266	1.000	.687	.094	.0625	.0625	.122
10265	.875	.500	.094	.051	.051	.090
10282	.750	.500	.0625	.040	.040	.057
3046	1.500	.5625	.075	.050	.050	.1154
5436	1.500	1.000	.156	.125	.094	.32152
12224	1.094	.625	.125	.1094	.1094	.20485
K-4200	1.094	.625	.1094	.0781	.0781	.15555
K-766	.875	.500	.094	.0625	.0625	.10399
12678	.500	.438	.0625	.040	.040	.04476

I. STATEMENT OF PROBLEM

Extrusions of various shapes riveted to sheet to give added stiffness and rigidity, are used extensively in aeronautical construction, gradually replacing sections made up formed flat sheet.

For this part of the research one of the extruded bulb angle sections which was investigated for its column properties in part one, was chosen for the stiffener to be riveted to the sheet.

The bulb angle section and the sheet which were used to make the panels, were 24ST alloy.

II. DESCRIPTION AND TABULATION OF PANELS

For the stiffener in the panel, bulb angle section Alcoa number 10282 with a cross-sectional area of .0647 square inches and a radius of gyration of .27 inches, was selected. For all panels the stiffener spacing was taken as 4 inches and the rivet spacing as .75 inches[(4)]. Two thicknesses of sheet were used, namely, .02 and .04 inches. Panels of two, three, and four stiffeners were made and tested, using the above spacings and thicknesses of sheet. In order to cover the current range of bulkhead spacings and at the same time extend into both the Euler long and short column ranges, the lengths of the panels chosen for test purposes were 3, $5\frac{1}{2}$, 11, $16\frac{1}{2}$, 22, and $27\frac{1}{2}$ inches. In each panel the sheet extended 2 inches beyond each outboard stiffener, that is, the width of the panel with two stiffeners was 8 inches; three stiffeners, 12 inches; four stiffeners, 16 inches.

The following tabulation describes the panels which were tested:

No. of stiff.	Sheet thick.	Stiff spac.	Panel len.	Panel width	Stiff. area	Sheet area	Total area	Total load	Average stress
2	.02	4	3	8	.1294	.16	.2894	6250	21600
2	.02	4	$5\frac{1}{2}$	8	.1294	.16	.2894	6550	22640
2	.02	4	11	8	.1294	.16	.2894	5310	18350
2	.02	4	$16\frac{1}{2}$	8	.1294	.16	.2894	6170	21300
2	.02	4	22	8	.1294	.16	.2894	6160	21300
2	.02	4	$27\frac{1}{2}$	8	.1294	.16	.2894	5040	17400
2	.04	4	3	8	.1294	.32	.4494	9460	21050
2	.04	4	$5\frac{1}{2}$	8	.1294	.32	.4494	11080	24700
2	.04	4	11	8	.1294	.32	.4494	9150	20360
2	.04	4	$16\frac{1}{2}$	8	.1294	.32	.4494	10080	22470
2	.04	4	22	8	.1294	.32	.4494	11150	24800
2	.04	4	$27\frac{1}{2}$	8	.1294	.32	.4494	9830	21900
3	.02	4	3	12	.1941	.24	.4341	7860	18100
3	.02	4	$5\frac{1}{2}$	12	.1941	.24	.4341	9390	21630
3	.02	4	11	12	.1941	.24	.4341	7675	17680
3	.02	4	$16\frac{1}{2}$	12	.1941	.24	.4341	8915	20530
3	.02	4	22	12	.1941	.24	.4341	7602	17500
3	.02	4	$27\frac{1}{2}$	12	.1941	.24	.4341	6650	15320
3	.04	4	3	12	.1941	.48	.6741	16170	23970
3	.04	4	$5\frac{1}{2}$	12	.1941	.48	.6741	12290	18230
3	.04	4	11	12	.1941	.48	.6741	12870	19090
3	.04	4	$16\frac{1}{2}$	12	.1941	.48	.6741	14010	20780
3	.04	4	22	12	.1941	.48	.6741	14938	22160
3	.04	4	$27\frac{1}{2}$	12	.1941	.48	.6741	12240	18150
4	.02	4	3	16	.2588	.32	.5788	11855	20500
4	.02	4	$5\frac{1}{2}$	16	.2588	.32	.5788	11450	19780
4	.02	4	11	16	.2588	.32	.5788	8720	15060
4	.02	4	$16\frac{1}{2}$	16	.2588	.32	.5788	11390	19670
4	.02	4	22	16	.2588	.32	.5788	9985	17260
4	.02	4	$27\frac{1}{2}$	16	.2588	.32	.5788	8810	15220
4	.04	4	3	16	.2588	.64	.8988	18200	20250
4	.04	4	$5\frac{1}{2}$	16	.2588	.64	.8988	20230	22500
4	.04	4	11	16	.2588	.64	.8988	17120	19050
4	.04	4	$16\frac{1}{2}$	16	.2588	.64	.8988	18590	20670
4	.04	4	22	16	.2588	.64	.8988	17938	19960
4	.04	4	$27\frac{1}{2}$	16	.2588	.64	.8988	14825	16500

III. TESTING PROCEDURE

In tests of this kind it is imperative that the opposite ends be parallel in order to have an even distribution of the load. With this in mind the panels were fabricated with a plus allowance in each length, and then placed in the milling machine and milled to the lengths chosen for test purposes.

Before the panel was put in the testing machine, two extensometers were placed on the side of the sheet opposite side to which the stiffeners were attached, and near the point of attachment of the end stiffeners of the panel. From the readings of these extensometers the effective width of the sheet acting with the stiffener can be computed, giving a check on other methods.

Even though the panels were made so that the ends were as nearly parallel as possible, it was found to be necessary, particularly in the wider panels, to shim the ends in order to obtain an even distribution of the load. When the panel was placed in the testing machine a slight load was applied and, if necessary, shims were inserted until it seemed apparent that the load was evenly distributed along the width of the panel. The load was then increased until the panel failed. In addition to the extensometer readings and failure load, the general behavior of each panel was recorded, noting in particular the first appearance of waves in both sheet and stiffener and the passage of waves through the rivet spacing. The latter effect was not pronounced or consistent and failed, in many cases, to leave a permanent set in the rivet line.

IV. CALCULATIONS

In order to plot the Euler curves, (Figs. 1,2,3), it is necessary to know the amount of sheet theoretically acting with the stiffener and the end load carried by the end tubes. The total Euler load will then be given by the formula,

$$P = n \times A_{(\text{stiffener} + \text{effective sheet})} \times \sigma + 2P_{end}$$

P_{end} is obtained readily from the curve giving the experimental "faired" values of P, (Fig.6). In order to obtain consistent results a cross-plot was made. First values of P were plotted vs. number of stiffeners, (Figs. 4,5), then from the faired curves a second plot was made of P vs. length. These curves are included in the appendix, (Fig.6).

As an example P_{end} will be calculated for the $27\frac{1}{2}$ inch, 0.020 thickness panel.

$$P_2 = 2P_{s+s} + 2P_{end}$$

$$P_3 = 3P_{s+s} + 2P_{end}$$

$$P_4 = 4P_{s+s} + 2P_{end}$$

where subscript s+ s refers to stiffener plus effective sheet.

$$P_2 = 4800 = 2P_{s+s} + 2P_{end}$$

$$P_3 = 6850 = 3P_{s+s} + 2P_{end}$$

$$-2050 = -P_{s+s}$$

$$P_{s+s} = 2050 \text{ lbs.}$$

$$P_{end} = \frac{4800 - 4100}{2} = 350 \text{ lbs.}$$

This method is followed for all panel lengths, sheet thicknesses, and stiffener combinations. Since there will be some scatter a curve was faired, through the plotted values of P_{end}, (Fig.7), and the mean values used to find the Euler load.

It is now necessary to determine the Euler stress (σ_E) of the stiffener alone, for two values of end fixity, i.e. k = 2 and k = 3.

$$\sigma_E = \frac{k\pi^2 E}{(L/\rho)^2}$$

$$\rho = 0.27$$

Tabulating the results, we obtain:

L	L/ρ	$(L/\rho)^2$	$2\pi^2 E$	$3\pi^2 E$	$\sigma_E(K=2)$	$\sigma_E(K=3)$
27.5	102	10400	207,300,000	310,500,000	19,900	29,800
22.0	81.5	6640	207,300,000	310,500,000	31,100	46,800
16.5	61.0	3720	207,300,000	310,500,000	55,600	83,500
11.0	40.8	1665	207,300,000	310,500,000	124,000	186,400
5.5	20.4	416	207,300,000	310,500,000	497,000	746,000
3.0	11.1	123	207,300,000	310,500,000	1,680,000	2,520,000

We can compute constants for use in the two curves drawn by Dr. E. E. Sechler of the Institute, (Figs. B-2,B-3, taken from reference (3)).

$$\lambda = t/b\sqrt{E/\sigma} = \frac{3235t}{b}\sqrt{1/\sigma}$$

for 0.020" sheet

$$\lambda = \frac{16.2}{\sigma^{\frac{1}{2}}}$$

$t/A_o = 0.309$

for 0.040" sheet

$$\lambda = \frac{32.4}{\sigma^{\frac{1}{2}}}$$

$t/A_o = 0.618$

where A_o = 0.0647 (for 10282 stiffener)

$S/\rho_o = 1.15$ (0.020" sheet)

$S/\rho_o = 1.20$ (0.040" sheet)

The method of using the curves will be shown by working one length.

$L = 27.5''$

$t = 0.020''$

$K = 2$

σ_E (from preceding page) $= 19900$ lbs/in^2

$\sigma^{\frac{1}{2}} = 141$

$$\lambda = \frac{16.2}{141} = 0.115$$

Entering curve of effective widths, (Fig. B-2); in this case we are on the transition curve between A and B.

$W_{e/b} = 0.265$ $\qquad$ $b = 4$ (stiffener spacing)

$2W_e = 2.12$

From Fig. B-3 we obtain

$\sigma/\sigma_o = 0.92$

$\sigma_1 = 19900 \times .92 = \underline{18300}$ lbs/in^2

$\sigma_1^{\frac{1}{2}} = 135$

$\lambda = 0.12$

$W_{e/b} = 0.265$

$2W_e = \underline{2.12}$

$\sigma/\sigma_o = 0.92$

The correct values of σ and $2W_e$ are therefore 18300 lbs/in^2 and 2.12 inches. We know this to be approximately correct since the ratio σ/σ_o remains the same. The tabulated results can be found in Table II Appendix.

The theoretical values of P can now be calculated using the curves of P_{end} vs. length, (Fig. 7 and table II).

Sample Calculation:

$L = 27.5$ inches $\quad t = 0.020"$ $\quad \sigma = 18300$ lbs./in^2

$A_{stiffener} = 0.0647$ $\quad 2W_e = 2.12"$ $\quad n$ = number stiffeners = 2

$$A_{s+s} = 0.0647 + 2.12 \times 0.02 = 0.1071$$

$$P = n \times A_{s+s} \times \sigma + 2P_{end}$$

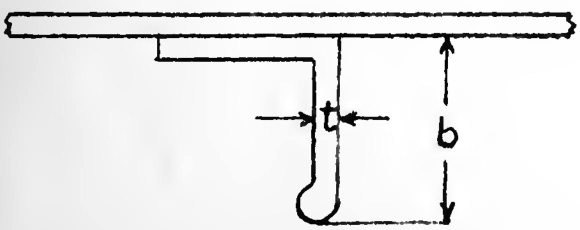
$$P = 2 \times 0.1071 \times 18300 + 2 \times 230$$

$$P = \underline{4380 \text{ lbs}}.$$

Plate Failure of the Outstanding Stiffener Leg.

In discussing this subject we shall follow the method of Timoshenko (Vol. II, Strength of Materials) and will attempt to determine a value of K which will suit the conditions of support of the panels tested. We know that none of the conditions of support for which Timoshenko has tabulated values of K exactly fit our case. In our panels we have three sides in which the fixity varies between a simple support and a built-in condition and a fourth side which is not entirely free due to some restraint provided by the bulb of the bulb angle section.

From our results we will determine an experimental value of K.



Length of Panel = a
b = 0.75
t = 0.047

$$\sigma_{cr} = K\,\sigma_e$$

$$\sigma_e = \frac{\pi^2 E t^2}{12 b^2 (1-\mu^2)} = \frac{\pi^2 \times 10{,}500{,}000 \times (0.047)^2}{12 \times (0.75)^2 \times (1-.09)}$$

$$\sigma_e = \frac{228{,}000}{6.14} = 37150 \text{ lbs/in}^2$$

It must be noted at this point that σ_{cr} is the actual critical stress in the stiffener leg. It will probably always be difficult to predict this stress from the average stress in the panel or from the compressive load and the geometry of the cross-section. We have, therefore, defined σ_{cr} as the average stress in the panel at failure.

$$\sigma_{cr} = \frac{P_{failure}}{A_{total}}$$

It is felt that this definition will permit the engineer to work entirely with known quantities: An experimental value of K will now be determined, using the properties of the three-stiffener section. This K will be tabulated for different values of a/b and checked by predicting the failure load for several of the panels with two and four stiffeners.

<u>Method of obtaining K:</u>

(a) <u>3 - Stiffeners</u>

$L = a = 3''$ $\quad b = 0.75''$ $\quad a/b = 4.0$

thickness sheet = 0.020"

$\sigma_e = 37150 \text{ lbs/in}^2$

$A_{total} = 3\,A_{stiffener} + A_{sheet} = 3 \times .0647 + .02 \times 12$

$A_t = 0.4341$

$P = 8700$

$\sigma_{cr} = \frac{8700}{.4341} = 20050 \text{ lbs/in}^2$

$K = \frac{20050}{37150} = 0.540$

(b) 3 - Stiffeners

$a = 3''$ $b = 0.75''$ $a/b = 4.0$

thickness sheet = 0.040"

$\sigma_e = 37150 \text{ lbs/in}^2$

$A_t = 3 \times .0647 + .04 \times 12$ $P = 14700$ lbs.

$\sigma_{cr} = \frac{14700}{.6741} = 21800 \text{ lbs/in}^2$

$K = \frac{21800}{37150} = \underline{0.587}$

And similarly for other values of a/b.

Table of K for various values of a/b

thickness of sheet - 0.020"

a/b =	4.0	7.35	14.68	22.0	29.35	36.70
K =	0.540	0.537	0.533	0.521	0.502	0.446

thickness of sheet - 0.040"

a/b =	4.0	7.35	14.68	22.0	29.35	36.70
K =	0.587	0.585	0.582	0.575	0.546	0.488

The reason for this difference in K with varying sheet thicknesses is probably due to the increase in end fixity with increasing sheet thickness.

It should now be possible to use these values of K and predict σ_{cr} and hence the load at failure. We will do this as a check on K by working out a theoretical P for several panels with two and four stiffeners, comparing the value obtained with the actual load at failure.

It should be noted that σ_e is a constant for any given stiffener. In the case of the 10282 section used in this investigation, $\sigma_e = 37150$ lbs/in^2.

<u>Checking values of K</u>:

(a) 2 - Stiffener, sheet thickness 0.020"

$a = 16.5"$ $\quad b = 0.75"$ $\quad a/b = 22.0$

$K = 0.521$

$\sigma_e = 37150$ lbs/in^2

$\sigma_{cr} = K\sigma_e = 0.521 \times 37150 = 19350$ lbs/in^2

$A_t = 2 \times 0.647 + .02 \times 8 = .2894$

$P_{theory} = \sigma_{cr} A_t = 19350 \times .2894 = \underline{5610 \text{ lbs}}$.

$P_{actual} = \underline{6170 \text{ lbs}}$.

(b) 2 - Stiffener, sheet thickness 0.040

$a = 5.5"$ $\quad b = 0.75"$ $\quad a/b = 7.35$

$K = 0.585$

$\sigma_e = 37150$ lbs/in^2

$\sigma_{cr} = 0.585 \times 37150 = 21700$ lbs/in^2

$A_t = 2 \times .0647 + .04 \times 8 = .4494$

$P_{theory} = .4494 \times 21700 = \underline{9760 \text{ lbs}}$.

$P_{actual} = \underline{11080 \text{ lbs}}$.

Checking values of K:

(c) 4 - Stiffeners, sheet thickness = 0.020

$a = 3"$ $b = 0.75"$ $a/b = 4.0$

$K = 0.54$

$\sigma_e = 37150 \text{ lbs/in}^2$

$\sigma_{cr} = K\sigma_e = 20050 \text{ lbs/in}^2$

$A_t = 4 \times .0647 + .02 \times 16 = .5788$

$P_{theory} = \sigma_{cr} A_t =$ <u>11600 lbs.</u>

$P_{actual} =$ <u>11855 lbs.</u>

(d) 4 - Stiffeners, sheet thickness = 0.040"

$a = 22"$ $b = 0.75"$ $a/b = 29.35$

$K = 0.546$

$\sigma_e = 37150 \text{ lbs/in}^2$

$\sigma_{cr} = 20300 \text{ lbs/in}^2$

$A_t = 4 \times .0647 + .04 \times 16 = .8988$

$P_{theory} =$ <u>18200 lbs.</u>

$P_{actual} =$ <u>17938 lbs.</u>

From our results in general it is believed that these values of K will give conservative results.

V. DISCUSSION OF EXPERIMENTAL RESULTS

The behavior of the panel while under test was interesting. The formation of waves, while the same in both thicknesses of sheet, was much easier to see in the 0.020" panels. Under a relatively low load a slight wave is first noticed in the sheet between stiffeners. As the load is increased the waves in the sheet become deeper and extend closer to the stiffeners, while the outstanding leg of the stiffeners goes into a wave form. Near failure the waves in the sheet go through the rivet line of the stiffeners and the waves in the outstanding leg become pronounced. In every panel tested failure resulted from a plate failure of the outstanding leg of the stiffener. In the 27.5" panels a tendency was noted for the panel to fail as an Euler column, however, the critical condition was still a plate failure of the outstanding stiffener leg.

From the curves, (fig. 1,2,3) the actual end fixity was estimated. This was done by plotting Euler curves for different values of fixity and observing where the faired experimental curves became tangent to the Euler curves. The end fixity of the panels tested was: 2.7 for the 0.020 sheet thickness and 3.2 for the 0.040 sheet thickness.

From our investigation it would appear that for the particular combination of sheet, stiffener and rivet spacing used an empirical value of k can be determined. This value of k is used in the formulae given by Timoshenko (Strength of Materials, Vol. II, page 605). The definition of σ_{cr} as the

average stress in the panel at failure is, we believe, justified since it is that stress which the designer will calculate in any application to stressed skin structures. Since σ_e is a constant for a particular stiffener; it is only necessary to look up k, entering table IV with the ratio

$$a/b = \frac{\text{rib or bulkhead spacing}}{\text{height of outstanding leg}}$$

The value of σ_{cr} may then be calculated from the formula:

$$\sigma_{cr} = k\sigma_e$$

If the average stress (using whole area) is equal to or above σ_{cr} the panel will fail due to plate failure of the outstanding leg of the stiffener. It is suggested that more work be done along this line, using other stiffener, sheet combinations, and attempting to achieve an end fixity which will closely approximate those found in the actual structure of the airplane.

VI. CONCLUSIONS

(1) It will be evident from an examination of Figures 1, 2, and 3, that the experimental values for the eleven inch panels fall considerably below the faired curves. This error is consistent, and is present in all combinations of sheet thicknesses and number of stiffeners tested in this length. There is no explanation for this behavior and lack of time prevented a re-check by the authors. It is recommended that the eleven inch points be regarded with doubt and checked by the personnel assigned to this research next year.

(2) The end fixity for the panels tested was found to be:

2.7 (sheet thickness of 0.020")
3.2 (sheet thickness of 0.040")

(3) For the type of bulb angle section used as stiffener in these tests an experimental value of K has been determined, (table IV). The proper value of K, taken from table IV, is to be used in Timoshenko's formulae (Strength of Materials, Vol. II, page 605), with modifications as outlined in Section IV of this paper. Following the method given, a close approximation will be obtained for the load at which plate failure of the outstanding leg occurs.

(4) There was no tendency for the type stiffener used in these tests to fail in torsion.

(5) Extensometer readings were taken during all tests. From these readings the theoretical value of the effective width of sheet acting with the stiffener could be checked. The authors were unable to carry this project through in the time available and it is suggested that it be done at a later date by the group assigned to a like problem next year.

(6) It is recommended that this project be carried on following similar lines. It would be desirable to test at least two panels in each length, sheet thickness, and number of stiffeners. The eleven inch lengths in the panels tested by the authors should again be tested and a check made of the values obtained in this investigation. Due to the great importance of this type of construction in the Industry it is suggested that as many of the remaining eleven bulb angle sections (see Part One) be attached to sheet and tested as time will allow.

(7) An important variable which will affect the value of K obtained for the outstanding leg and which should also be investigated is the ratio of the bulb area to the area of the outstanding leg of the section. This ratio will determine to a large extent the amount of fixity to be assigned to the outstanding leg and therefore determine to a large extent the value of the constant K.

APPENDIX

REFERENCES

(1) Parr-Beakley Thesis of 1935.

(2) S. Timoshenko - Strength of Materials, Vol.II, page 606.

(3) Compressive Stress Distribution of Stiffened Sheet Panels, Dr.E. E. Sechler, Journal of the Aeronautical Sciences, June, 1937.

(4) Effect of Rivet Spacing on Stiffened Thin Sheet Under Compression, W. L. Howland, Journal of the Aeronautical Sciences, October, 1936.

Specimen	I_x'	A	ρ^2	ρ	L_{eff}	$\left(\frac{L_{eff}}{\rho}\right)$	$\left(\frac{L_{eff}}{\rho}\right)^2$	P_{Euler}	P_{actual}	σ_{theory}	σ_{actual}
10265	.00113	.0931	.01214	.110	24.24	220	48400	199	196	2120	2105
					18.73	170	28900	334	335	3590	3595
					13.21	120	14400	671	615	7210	6610
					7.62	69.3	4800	2020	1540	21700	16550
10282	.00087	.0647	.01343	.116	24.24	209	43700	153	147	2365	2275
					18.73	161.5	26080	256	210	3960	3250
					13.21	114	13000	515	410	7980	6340
					7.62	65.7	4310	1552	1025	24000	15870
3046	.00164	.1252	.01310	.1143	24.24	212	45000	289	294	2310	2345
					18.73	164	26900	484	405	3865	3240
					13.21	115.5	13350	973	700	7770	5600
					7.62	66.6	4440	2930	1945*	23400	15520*
5436	.01177	.2470	.0476	.218	24.24	111.2	12380	2070	2230	8390	9030
					18.73	85.9	7370	3470	3495	14070	14140
					13.21	60.6	3670	6980	6040	28300	24450
					7.62	35	1225	21000	9605	85000	38900
12224	.00348	.1709	.0204	.143	24.24	169.5	28750	613	820	3585	4800
					18.73	131	17150	1028	1180	6010	6900
					13.21	92.4	8520	2060	2320	12070	13590
					7.62	53.3	2840	6210	5840	36350	34200

\# Indicates a column failure of the bulb alone

Parameters are based on actual dimensions, not on those listed by the manufacturer.

Specimen	I_x'	A	ρ^2	ρ	L_{eff}	$\left(\frac{L_{eff}}{\rho}\right)$	$\left(\frac{L_{eff}}{\rho}\right)^2$	L_{eff}^2	P_{euler}	P_{actual}	σ_{theory}	σ_{actual}
8477	.01627	.2774	.0586	.242	24.24	100.2	10040	588	2865	2703	10330	9775
					18.73	75.7	5730	351	4800	4310	17300	15520
					13.21	54.7	2992	174.6	9640	7365	34750	26550
					7.62	31.5	992.3	58	29050	10370*	104800	37400*
8476	.00333	.1501	.0222	.149	24.24	162.4	26400	588	587	590	3910	3930
					18.73	125.7	15800	351	982	875	6540	5830
					13.21	88.7	7860	174.6	1975	1670	13170	11120
					7.62	51.1	2610	58	5940	4020	39550	26800
8478	.00877	.1679	.0521	.228	24.24	106.2	11290	588	1543	1490	9190	8870
					18.73	82.1	6740	351	2580	2275	15350	13540
					13.21	57.9	3355	174.6	5200	3875	31000	22450
					7.62	33.4	1114	58	15660	5770*	93200	34400*
10266	.00293	.1220	.0240	.155	24.24	156.3	24430	588	516	469	4240	3845
					18.73	120.8	14600	351	864	775	7080	6350
					13.21	85.2	7250	174.6	1737	1545	14250	12670
					7.62	49.1	2415	58	5223	2650	42800	21700

* Indicates plate failure

Parameters are based on actual dimensions, not on those listed by the manufacturer.

Specimen	I'_x	A	ρ^2	ρ	L_{eff}	$\left(\frac{L_{eff}}{\rho}\right)$	$\left(\frac{L_{eff}}{\rho}\right)^2$	P_{euler}	P_{actual}	σ_{theory}	σ_{actual}
4200	.00326	.1334	.0244	.156	24.24	155	24000	575	621	4305	4650
					18.73	120	14400	963	955	7210	7160
					13.21	84.8	7200	1933	1900	14500	14250
					7.62	48.9	2390	5820	4280	43600	32050
766	.00116	.0856	.0136	.1165	24.24	208	43250	204	260	2385	3040
					18.73	161	25950	342	385	4000	4500
					13.21	113.3	12850	688	705	8040	8240
					7.62	65.4	4280	2007	2005	23500	23450
12678	.000436	.04023	.0108	.104	24.24	233	54300	77	107	1923	2675
					18.73	180	32400	129	155	3220	3880
					13.21	127	16140	258	305	6450	7615
					7.62	73.3	5370	778	710	19450	17750

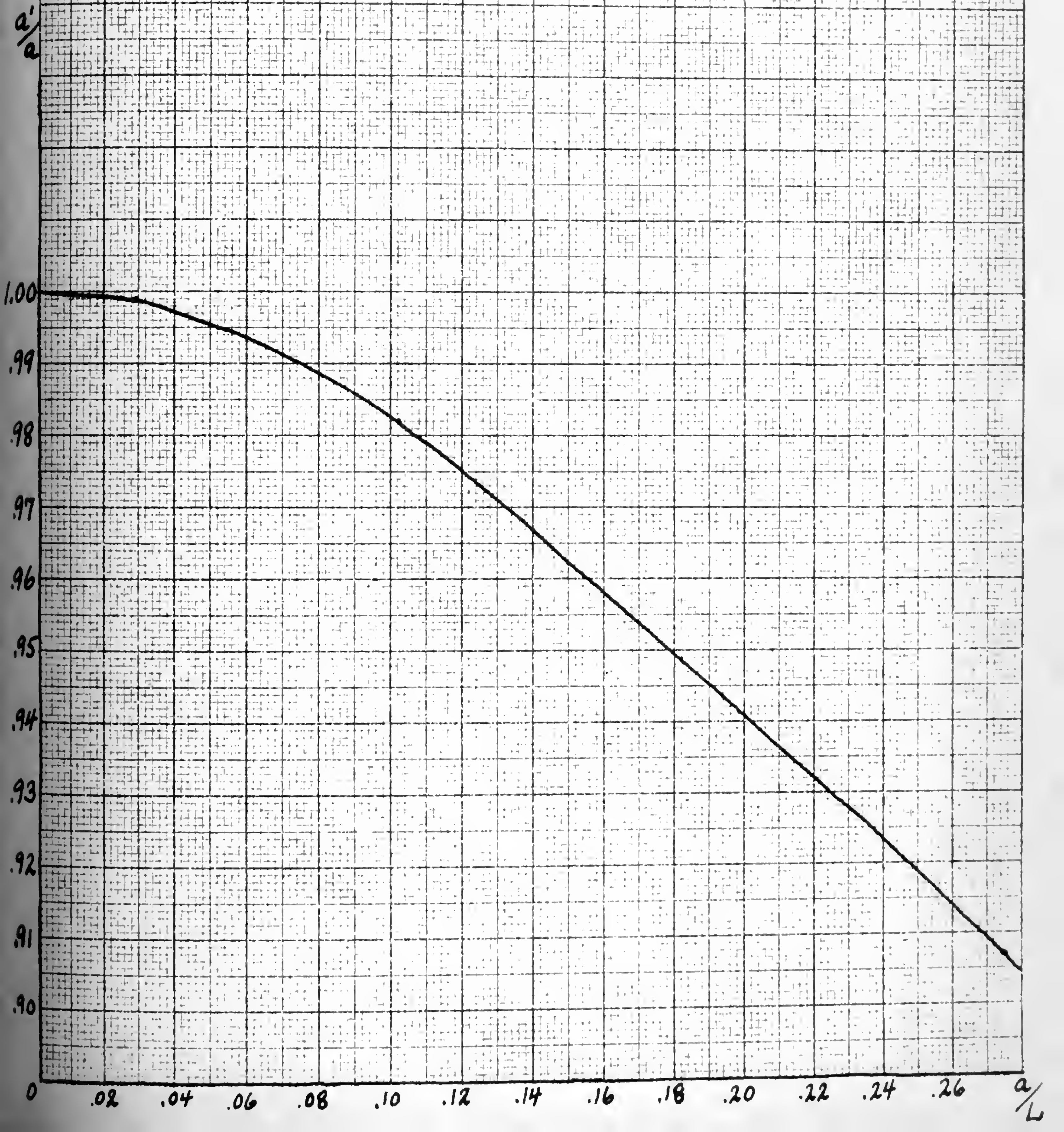
Parameters are based on actual dimensions, not on those listed by the manufacturer.

Part One

Fig. 1

To find L_{eff}.

Curve of a'/a vs a/L



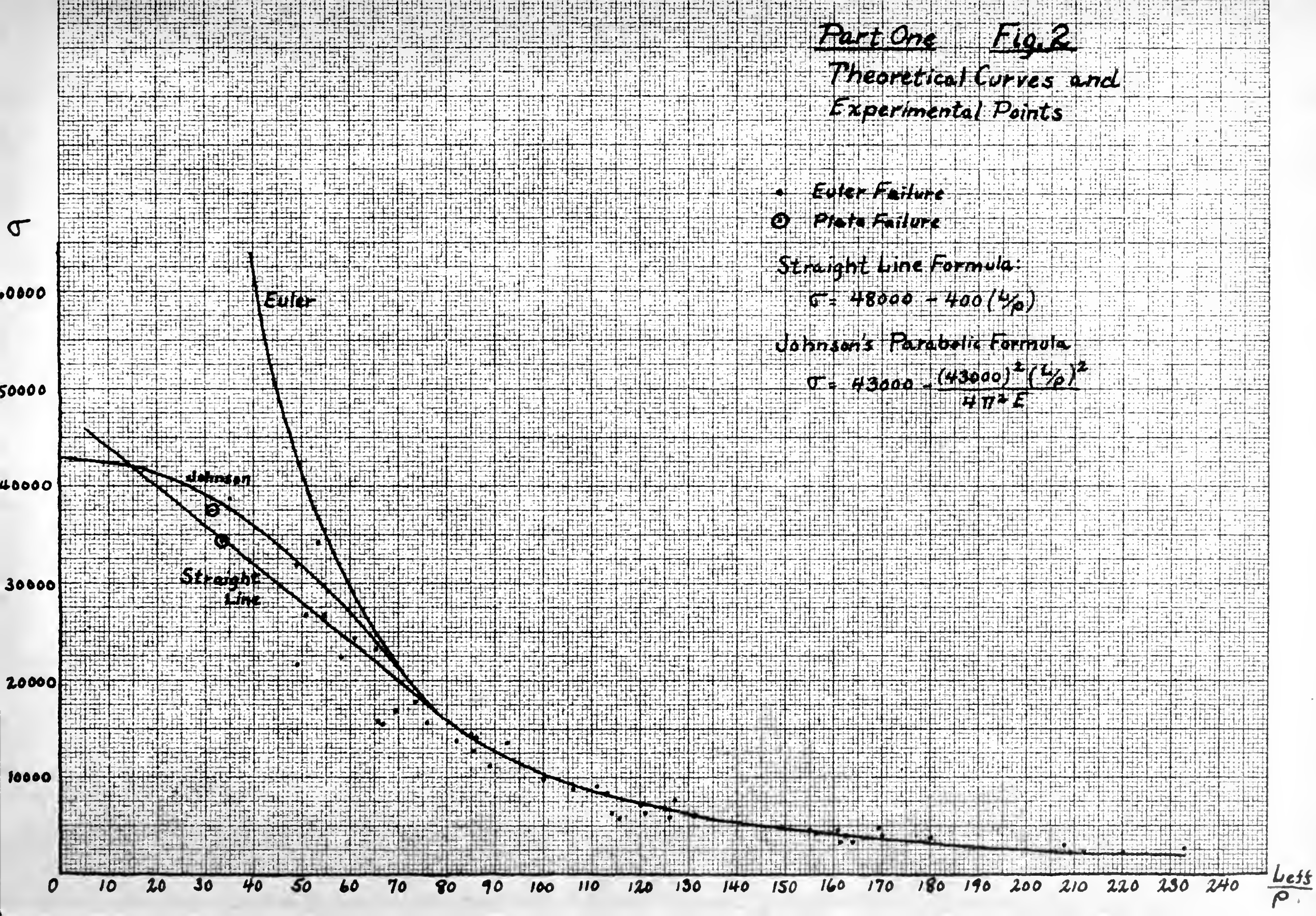

Part One Fig. 2
Theoretical Curves and
Experimental Points
Euler Failure
Plate Failure
Straight Line Formula:
σ = 48000 - 400 (L/ρ)
Johnson's Parabolic Formula
σ = 43000 - (43000)^2 (L/ρ)^2 / 4π^2 E
Euler
Johnson
Straight Line
σ
50000
40000
30000
20000
10000
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
Leff / ρ

Table I , Part Two

Calculation of Experimental $P_{stiffener + sheet}$ and P_{panel} from faired curves.

L	$P_{stiff. + sheet}$ 0.020"	$P_{stiff. + sheet}$ 0.040"	P_{panel} 0.020"	P_{panel} 0.040"	Avg. $P_{stiff. + sheet}$ 0.020"	Avg. $P_{stiff. + sheet}$ 0.040"	Avg. P_{panel} 0.020"	Avg. P_{panel} 0.040"
	2300	4000	0	100				
27.5	2100	3900	200	200	2100	3900	267	233
	1900	3800	600	400				
	2250	3800	625	1500				
22.0	2175	3800	700	1500	2175	3800	725	1500
	2100	3800	850	1500				
	2050	3800	1325	1500				
16.5	2125	3800	750	1500	2125	3800	1058	1500
	2200	3800	1100	1500				
	2200	3800	400	700				
11.0	2150	3850	450	650	2150	3850	467	633
	2100	3900	550	550				
	2100	3700	1500	1600				
5.5	2050	3800	1550	1500	2050	3800	1567	1467
	2000	3900	1650	1300				
	2200	3800	800	2100				
3.0	2100	3850	900	2050	2100	3850	933	2033
	2000	3900	1100	1950				

Table II, Part Two

Length	$k = 2$	$t = 0.020$	$k = 3$	$t = 0.020$	$k = 2$	$t = 0.040$	$k = 3$	$t = 0.040$
	σ	$2w_e$	σ	$2w_e$	σ	$2w_e$	σ	$2w_e$
27.5	18300	2.12	29800	1.24	17300	2.42	25600	1.84
22.0	31100	1.18	46800	0.496	26400	1.80	46800	0.88
16.5	55600	0.473	83500	0.384	55600	0.84	83500	0.704
11.0	124,000	0.32	186,400	0.28	126,400	0.60	192,000	0.498
5.5	497,000	0.176	746,000	0.144	511,000	0.32	769,000	0.264
3.0	1,680,000	0.08	2,520,000	0.064	1,715,000	0.178	2,570,000	0.144

The high stresses corresponding to the shorter lengths have no practical meaning, being points on the steep portion of the Euler curve.

Table III , Part Two

Lengths	$k = 2$ $t = 0.02$			$k = 3$ $t = 0.02$			$t = 0.04$ $k = 2$			$t = 0.04$ $k = 3$		
	P_2	P_3	P_4	P_2	P_3	P_4	P_2	P_3	P_4	P_2	P_3	P_4
27.5	4380	6330	8330	5800	8460	11140	7270	10060	12850	8760	12310	15850
22.0	6940	9390	12430	8440	11930	15400	9930	13540	17150	12070	16740	21420
16.5	10370	14490	18320	14220	20240	26270	14180	19660	25090	18750	26520	34220
11.0	20110	28890	37790	28690	41790	54890	25940	37140	48440	35990	52240	68440
5.5	70540	104,640	138,400	103,600	154,100	204,600	82,980	122,700	162,200	119,480	177,280	235,180
3.0	225,200	336,700	447,700	334,700	502,700	667,700	249,700	372,200	495,700	365,700	546,700	728,700

These values are used to plot the Euler Curves.

The subscript on P indicates the number of stiffeners.

The high values of P in the shorter lengths have no practical meaning, being points on the steep portion of the Euler curve.

Part Two

Table IV

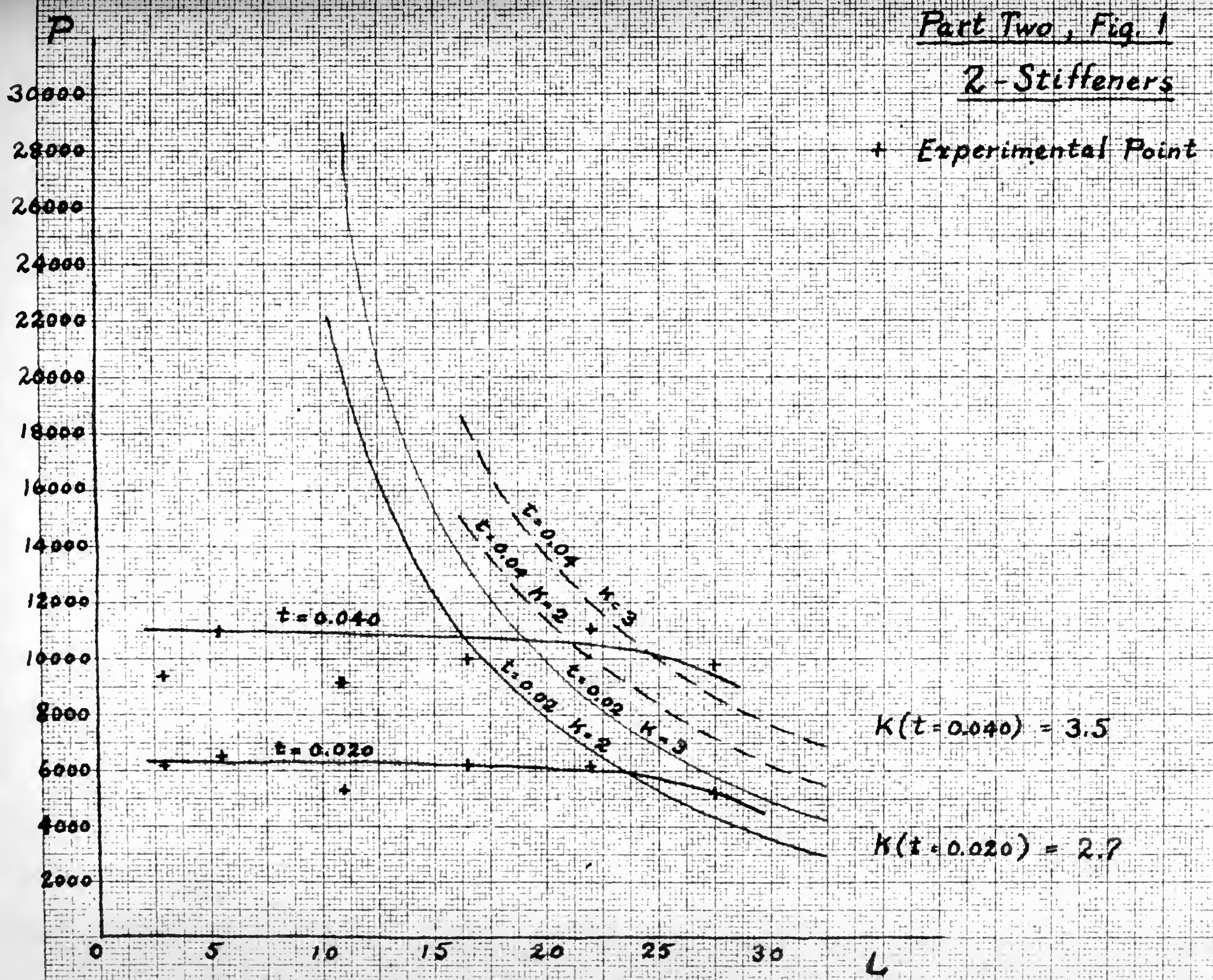
Thickness of sheet = 0.020"

a/b	4.0	7.35	14.68	22.0	29.35	36.70
K	0.540	0.537	0.533	0.521	0.502	0.446

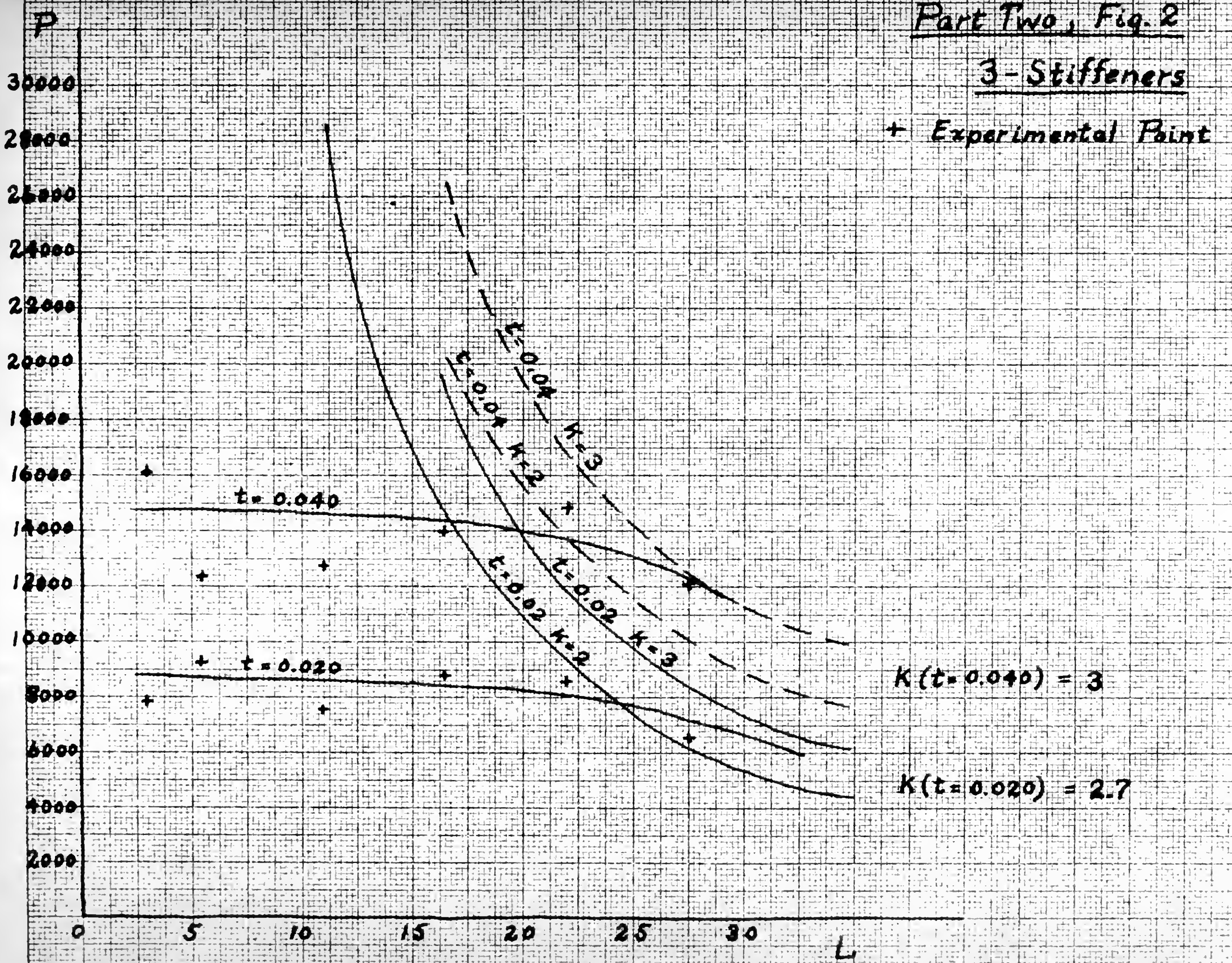
Thickness of sheet = 0.040"

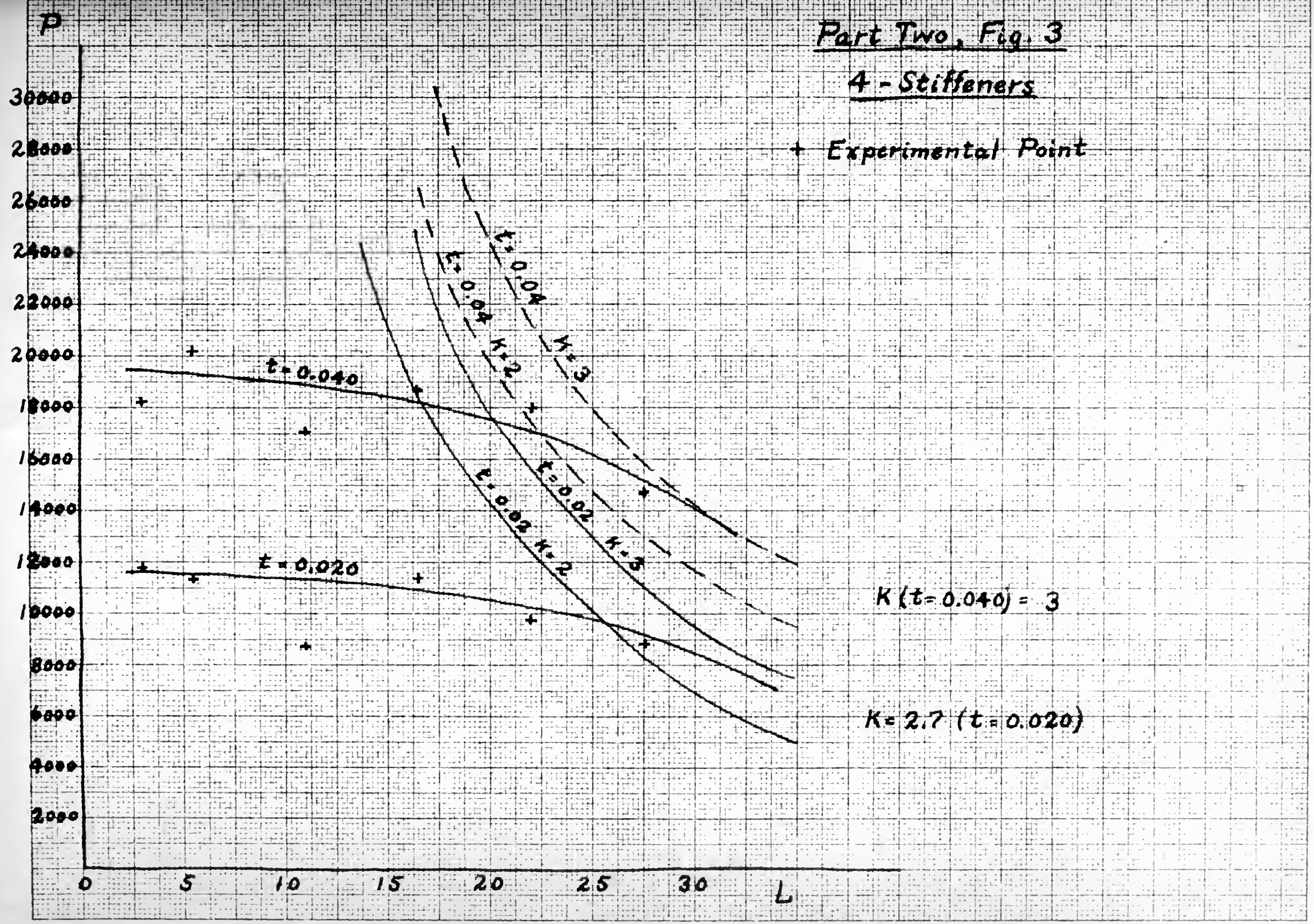
a/b	4.0	7.35	14.68	22.0	29.35	36.70
K	0.587	0.585	0.582	0.575	0.546	0.488

These values only apply when using ALCOA number 10282 extruded bulb angle sections as stiffeners.


Part Two, Fig. 1
2 - Stiffeners
+ Experimental Point
P
30000
28000
26000
24000
22000
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
5
10
15
20
25
30
L
t = 0.040
t = 0.020
t = 0.04 K = 3
t = 0.04 K = 2
t = 0.02 K = 3
t = 0.02 K = 2
K(t = 0.040) = 3.5
K(t = 0.020) = 2.7

Part Two, Fig. 2



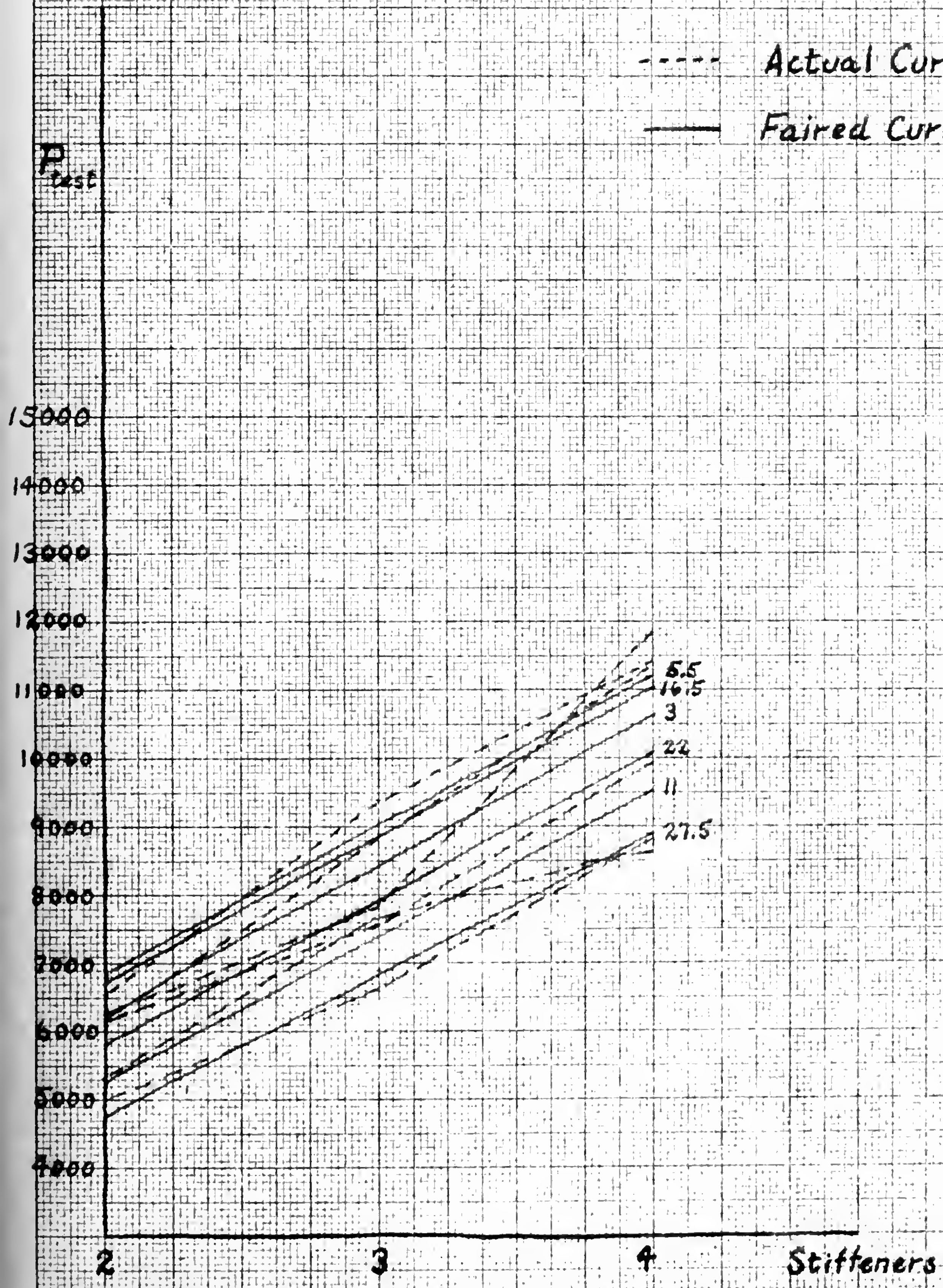

Part Two, Fig. 3
4 - Stiffeners
+ Experimental Point
K (t = 0.040) = 3
K = 2.7 (t = 0.020)
P
30000
28000
26000
24000
22000
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
5
10
15
20
25
30
L
t = 0.040
t = 0.020
t = 0.04 K = 3
t = 0.04 K = 2
t = 0.02 K = 3
t = 0.02 K = 2

Part Two, Fig. 4

Plot of P vs Number of Stiffeners

Used to determine mean P.

thickness = 0.020



Part Two, Fig. 5

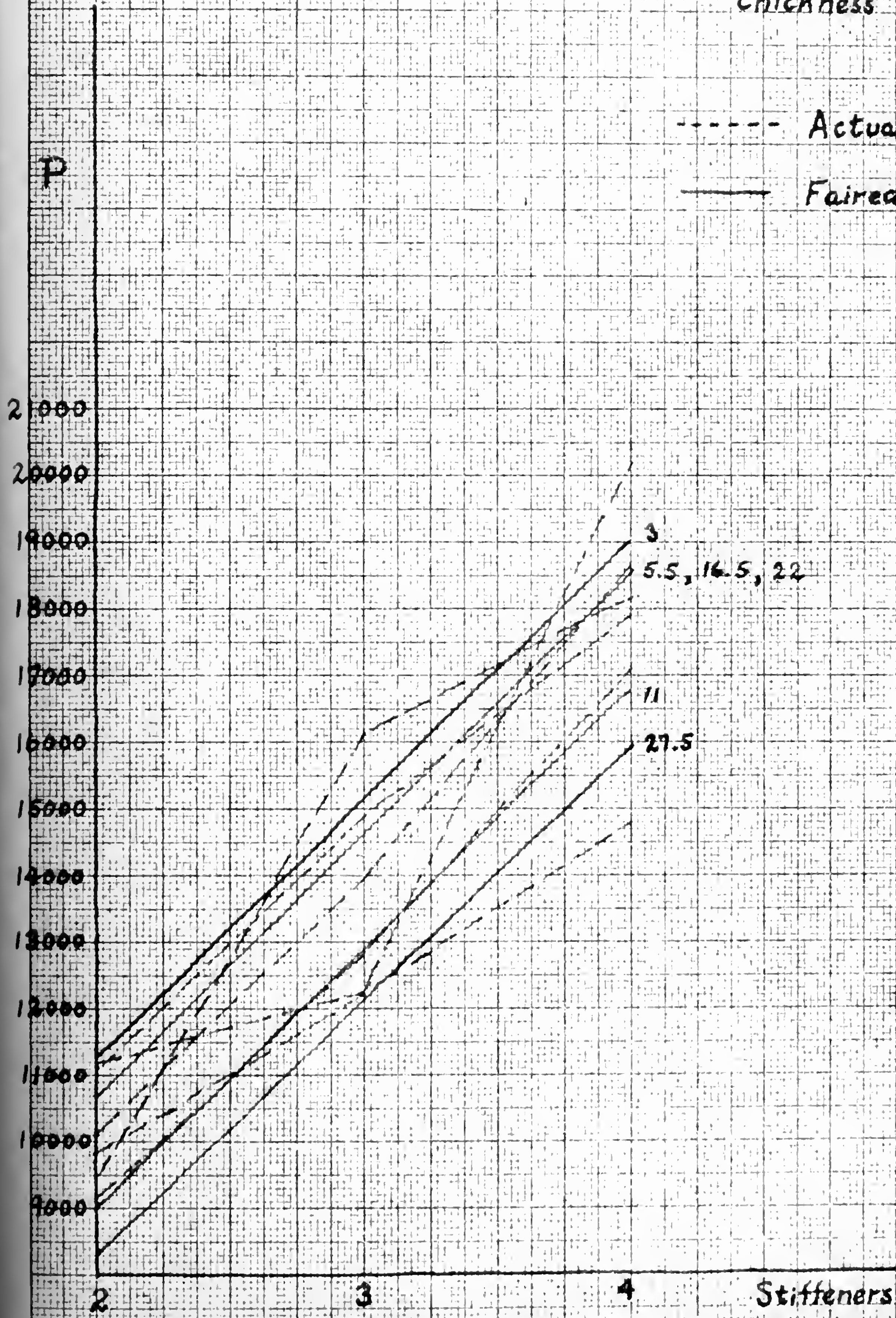
Plot of P vs Number of Stiffeners

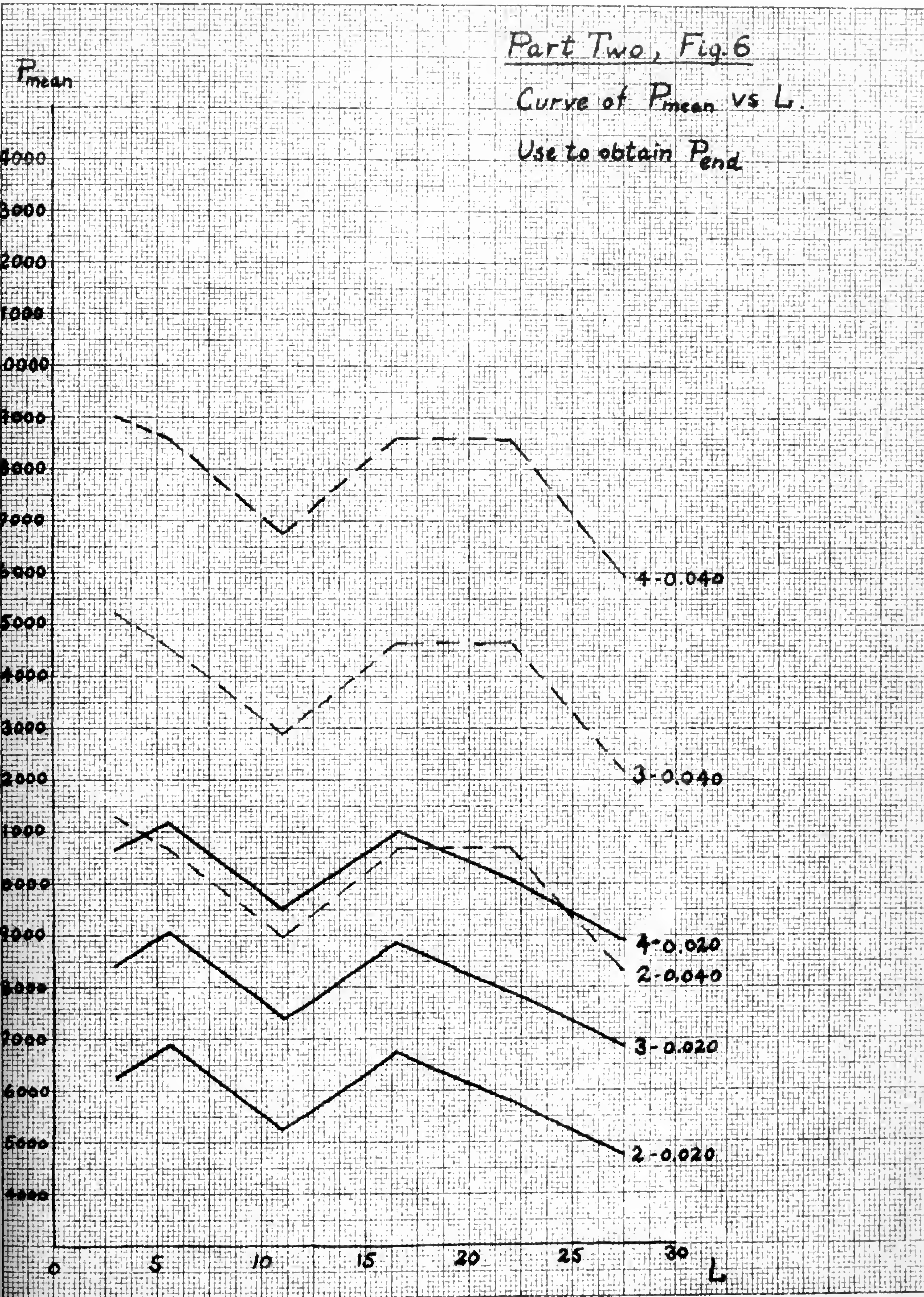
Used to determine mean P

thickness = 0.040

------ Actual Curves.

—— Faired Curves.

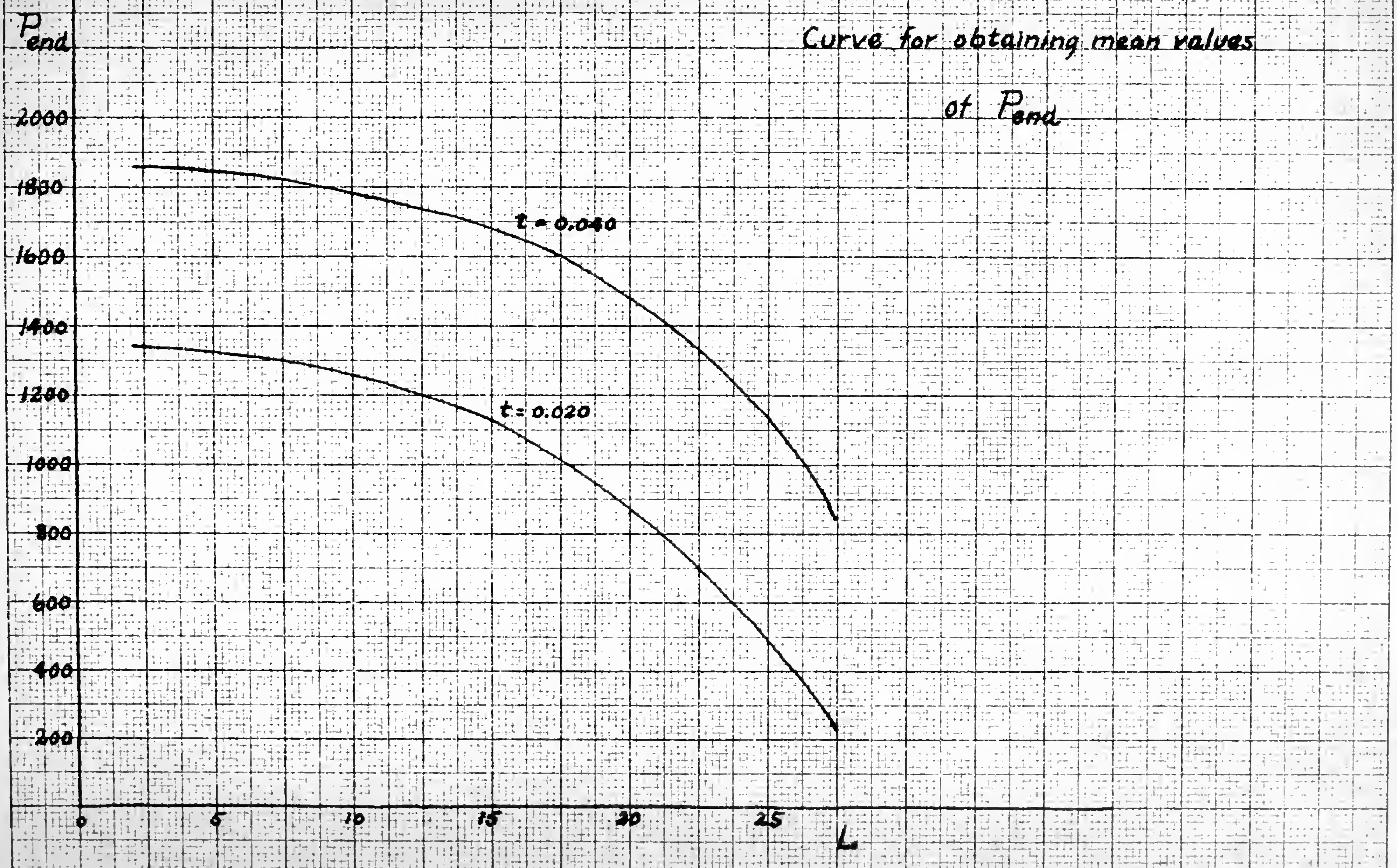




Part Two, Fig. 6
Curve of P_{mean} vs L.
Use to obtain P_{end}
P_{mean}
24000
23000
22000
21000
20000
19000
18000
17000
16000
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
0
5
10
15
20
25
30
L
4-0.040
3-0.040
4-0.020
2-0.040
3-0.020
2-0.020

Part Two, Fig. 7

Curve for obtaining mean values of P_{end}



EFFECTIVE WIDTH AS A FUNCTION OF λ.

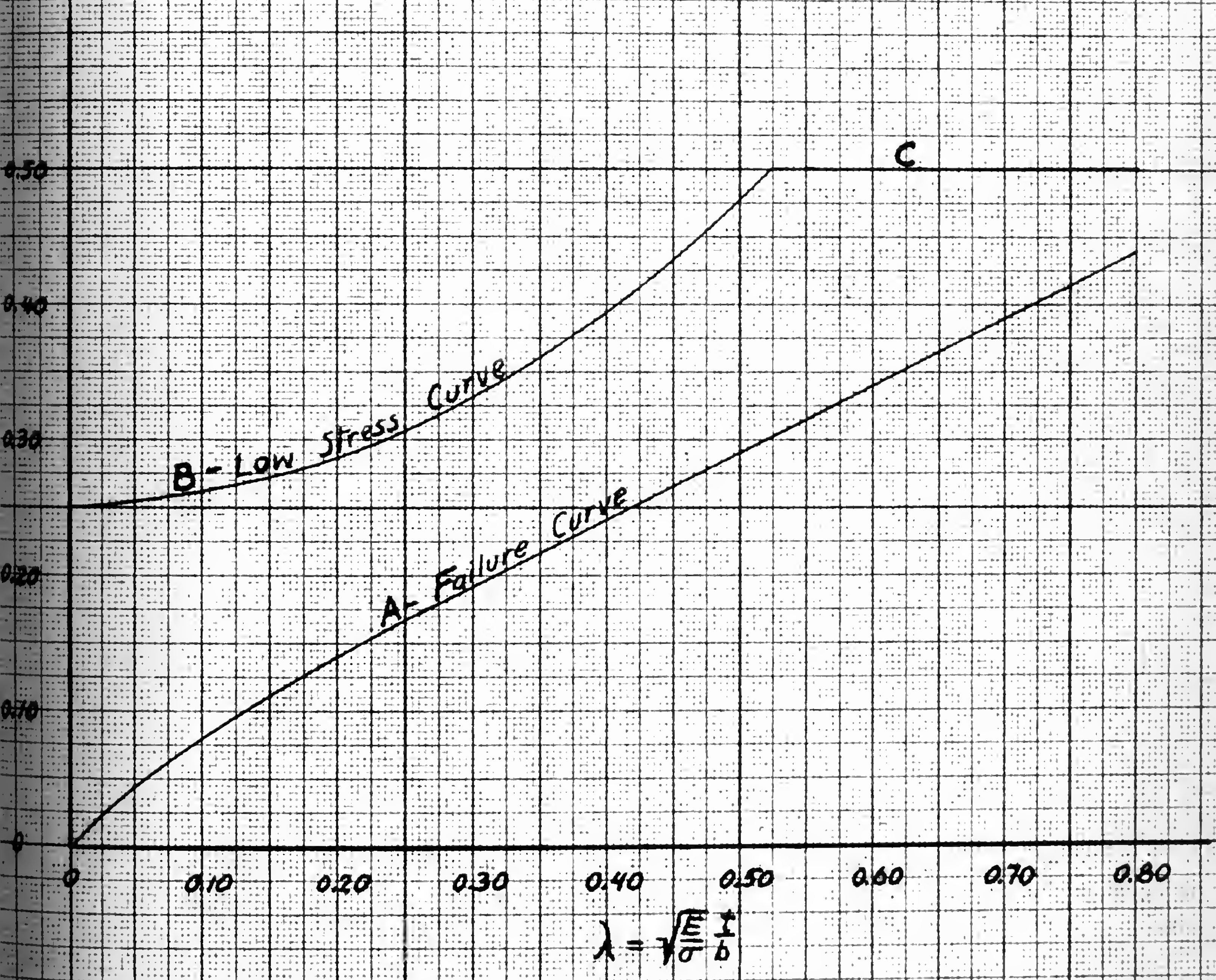


Fig. B-2

2-1-37

CURVE FOR THE DETERMINATION OF $\left(\frac{\rho}{\rho_o}\right)^2 = \frac{\sigma}{\sigma_o}$

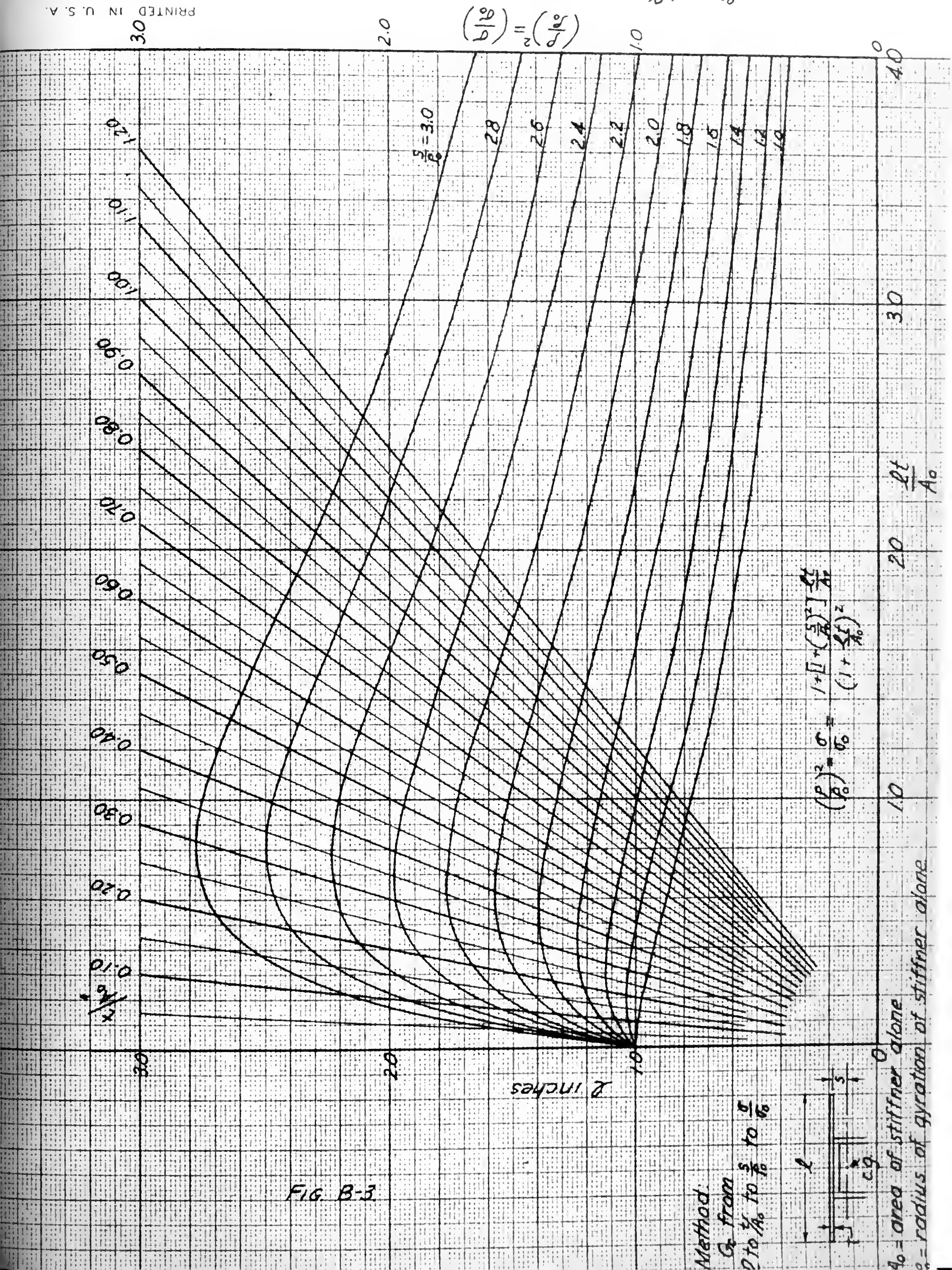
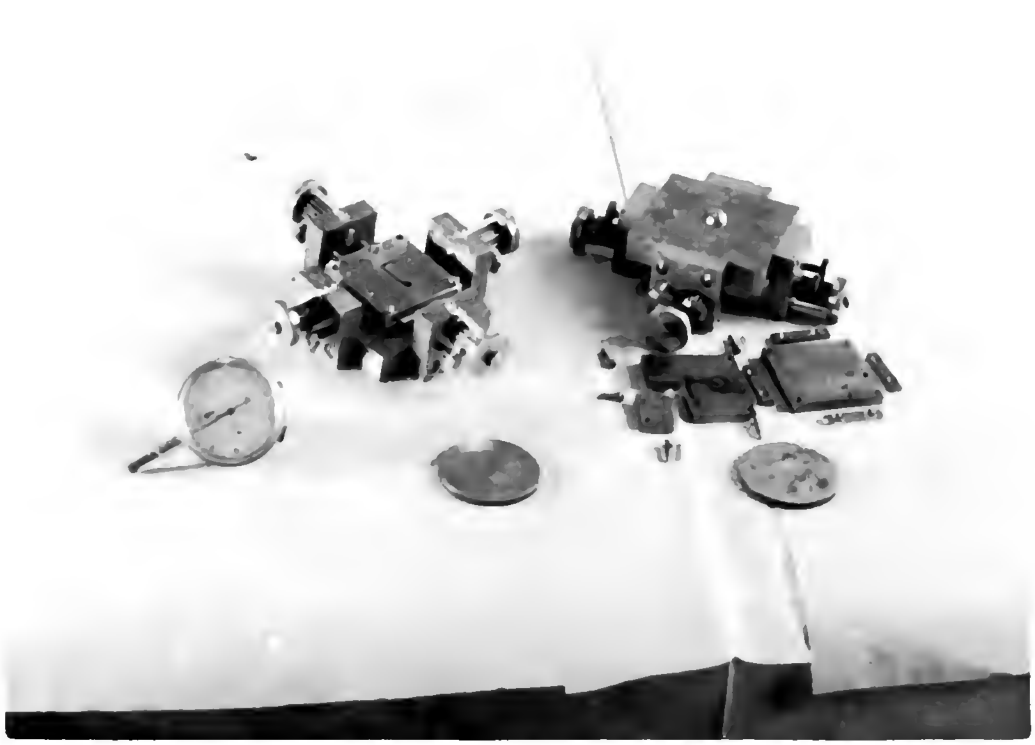
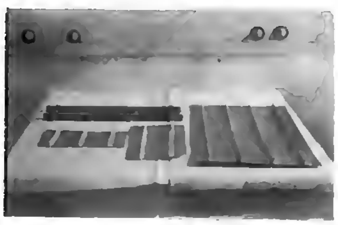
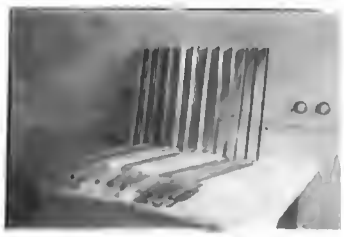
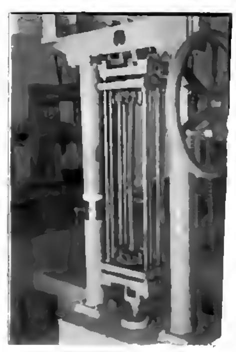


FIG. B-3









DATE DUE
